2/16/17

1.8.1 (a)
(b)
(c)

*1.8.2 (a) ✓
(c) ✓
(f) ✓

1.8.4

1.8.5 ✓

1.8.7

1.8.10 –

1.8.12

1.8.15 (a) –

1.8.18

1.8.22 (a)
(c)
(e)

1.8.24 ✓

*1.8.26 ✓

1.8.27 (a)
(b) –
(c)

1.8.2 (a) Determine if the following systems are compatible, and, if so, find the general solution:

$$6x_1 + 3x_2 = 12$$
$$4x_1 + 2x_2 = 9$$

$$\left[\begin{array}{cc|c} 6 & 3 & 12 \\ 4 & 2 & 9 \end{array}\right] \xrightarrow{-2/3 R_1 + R2} \left[\begin{array}{ccc} 6 & 3 & 12 \\ 0 & 0 & 1 \end{array}\right]$$

Not compatible

(c)

$$x_1 + 2x_2 = 1$$
$$2x_1 + 5x_2 = 2$$
$$3x_1 + 6x_2 = 3$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 5 & 2 \\ 3 & 6 & 3 \end{array}\right] \xrightarrow[-3R1+R3]{-2R1+R2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow$ $\boxed{x_2 = 0}$

$x_1 + 2(0) = 1$

$\boxed{x_1 = 1}$

(f)

$$x_1 + x_2 + x_3 + 9x_4 = 8$$
$$x_2 + 2x_3 + 8x_4 = 7$$
$$-3x_1 + x_3 - 7x_4 = 9$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 9 & 8 \\ 0 & 1 & 2 & 8 & 7 \\ -3 & 0 & 1 & -7 & 9 \end{array}\right] \xrightarrow{3R1+R3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 9 & 8 \\ 0 & 1 & 2 & 8 & 7 \\ 0 & 3 & 4 & 20 & 33 \end{array}\right]$$

$$\xrightarrow{-3R2+R3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 9 & 8 \\ 0 & 1 & 2 & 8 & 7 \\ 0 & 0 & -2 & -4 & 12 \end{array}\right]$$

$\boxed{x_4 \quad \text{free}}$

$$-2x_3 - 4x_4 = 12$$
$$x_3 + 2x_4 = -6$$
$$\boxed{x_3 = -6 - 2x_4}$$

$$x_2 + 2(-6 - 2x_4) + 8x_4 = 7$$
$$x_2 - 12 + 4x_4 = 7$$
$$\boxed{x_2 = 19 - 4x_4}$$

$x_1 + (19 - 4x_4) + (-6 - 2x_4) + 9x_4 = 8$

$x_1 + 13 - 6x_4 + 9x_4 = 8$

$\boxed{x_1 = -5 - 3x_4}$

1.8.26 Find the solution to

$2x_1 + x_2 + x_3 - x_4 = 0$
$2x_1 - 2x_2 - x_3 + 3x_4 = 0$

Then solve replacing the RH side w/ a & b.

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -1 & 0 \\ 2 & -2 & -1 & 3 & 0 \end{array}\right] \xrightarrow{-R1+R2} \left[\begin{array}{cccc|c} 2 & 1 & 1 & -1 & 0 \\ 0 & -3 & -2 & 4 & 0 \end{array}\right]$$

x_3, x_4 free

$-3x_2 - 2x_3 + 4x_3 = 0$

$-3x_2 = 2x_3 - 4x_3$

$x_2 = -2/3\, x_3 + 4/3\, x_4$

$2x_1 + (-2/3\, x_3 + 4/3\, x_4) + x_3 - x_4 = 0$

$\Rightarrow 2x_1 + 1/3\, x_3 + 1/3\, x_4 = 0$

$2x_1 = -1/3\, x_3 - 1/3\, x_4$

$x_1 = -1/6\, x_3 - 1/6\, x_4$

Soln :
$$\begin{pmatrix} -1/6\, x_3 - 1/6\, x_4 \\ -2/3\, x_3 + 4/3\, x_4 \\ x_3 \\ x_4 \end{pmatrix}$$

1.8.5 For which values of b and c does

$x_1 + x_2 + bx_3 = 1$

$bx_1 + 3x_2 - x_3 = -2$

$3x_1 + 4x_2 + x_3 = c$ have

(a) no soln

(b) exactly 1 soln

(c) infinitely many solns

$$\left[\begin{array}{ccc|c} 1 & 1 & b & 1 \\ b & 3 & -1 & -2 \\ 3 & 4 & 1 & c \end{array}\right] \xrightarrow{\substack{-bR1+R2 \\ -3R1+R3}} \left[\begin{array}{ccc|c} 1 & 1 & b & 1 \\ 0 & 3-b & -1-b^2 & -2-b \\ 0 & 1 & 1-3b & c-3 \end{array}\right]$$

$$\xrightarrow{R2 \leftrightarrow R3} \left[\begin{array}{ccc|c} 1 & 1 & b & 1 \\ 0 & 1 & 1-3b & c-3 \\ 0 & 3-b & -1-b^2 & -2-b \end{array}\right] \xrightarrow{-3+b\ R2+R3} \left[\begin{array}{ccc|c} 1 & 1 & b & 1 \\ 0 & 1 & 1-3b & c-3 \\ 0 & 0 & -4+10b-4b^2 & -3c+7+bc-4b \end{array}\right]$$

$(-3+\ldots$

$= -3+\ldots$

$+ \ -1\ldots$

$-4\ldots$

$(-3+b)(c-\ldots$

$= -3c + 9 + \ldots$

$+ \ -2-b$

$-3c+7+\ldots$

$$= \left[\begin{array}{ccc|c} 1 & 1 & b & 1 \\ 0 & 1 & 1-3b & c-3 \\ 0 & 0 & -2(2b-1)(b-2) & -3c+7+bc-4b \end{array}\right]$$

$2-5b+2b^2$

(a) no soln : $-2(2b-1)(b-2) = 0$ and $-3c+7+bc-\ldots$

$\boxed{b = 1/2}$ or $\boxed{b = 2}$

and

$-3c+7+\frac{1}{2}c - 2 \neq 0$

$\Rightarrow -\frac{5}{2}c \neq -5$

$\Rightarrow \boxed{c \neq 2}$

$-3c+7+2c-8 \neq 0$

$\Rightarrow -c-1 \neq 0$

$\Rightarrow -c \neq 1$

$\Rightarrow \boxed{c \neq -1}$

(b) 1 soln $-2(2b-1)(b-2) \neq 0$

$\Rightarrow$ $b \neq 1/2$ and $b \neq 2$.

(c) many solns : $-2(2b-1)(b-2) = 0$ and

$-3c+7+bc-4b = 0$

$\Rightarrow$ $b = 1/2$ or $b = 2$

and $c = 2$ and $c = -1$.

1.8.24. Let U be upper triangular. Show that $U\vec{x} = 0$ has a nontrivial soln iff U has at least one zero on its diagonal.

Proof:

$\Rightarrow$ Assume $U\vec{x} = 0$ has a nontrivial soln.

Then U is singular.

$\Rightarrow$ $\det U = 0$.

But $\det U$ = product of diagonal entries for upper triangula matrices

So one of the diag. entries must be zero.

$\Leftarrow$ Assume one of the diag. entries is zero.

Then $\det U = 0$ (again since upper Δ)

So U is singular.

Thus $U\vec{x} = 0$ has a nontrivial soln.